

Head Transplants by MatriX

¹ 1	² 5	³ 6	⁴ 9	⁵ 5	⁶ 2	⁷ 7
⁸ 4	4	⁹ 1	6	¹⁰ 2	¹¹ 2	7
¹² 6	9	¹³ 3	8	4	1	1

Explanation

1. Let $1d=pqr$ entered as Pqr . 8a options are Prq, rPq, rqP entered as qrq, qPq, qqP .

2. 8a entry is square (11d clue) or leading to non-triangular 1dn. 8ac entry= $q?q$ allows 121, 484, 676 all producing invalid negative 3a. 8a= qqP options are 225 or 441. 225 gives negative 3a, so 8a=441. 11d=21

3. 1d triangular with central digit 4, either 741 or 946 entered as 141 or 146 leading to 3a= 300 (invalid 0 starts) or 295. Confirms $1d=946$, entry 146, 8a=641 with entry 441 and 3a=295 pre-entry. The lead digit situation is:

3-digit entries	3a	8a	10a	13a	1d	4d	7d	Available
Answer	2	6			9			1,4
Entry		4			1			2,6,9

4. 5d/9d options are 51/17 (10a duplicates lead 1), 52/13, 54/18 (10a duplicates lead 4), 55/11 (13a duplicates lead 1), 56/14 (13a duplicate lead 4), 57/19 (4d cannot be multiple of 13a starting 9). Confirms $9d=13, 5d=52$ extending table to:

3-digit entries	3a	8a	10a	13a	1d	4d	7d	Available
Answer	2	6			9			1,3,4
Entry		4	2	3	1			6,9

5. 4d is twice 13a (three times gives duplicate 9 start). 4d cannot start with duplicate 6 so starts 7. 13a entry starts 3 and has central digit 6 or 8. 13a/4d possibilities are 363/726, 368/736, 384/768, 389/778. Pre-entry 13a is a square starting 1 or 4, and has to be 484.

3-digit entries	3a	8a	10a	13a	1d	4d	7d	Available
Answer	2	6		4	9	7		1,3
Entry		4	2	3	1	9		6,7

6. $10a=square-12a$. 10a starts 12 or 32, 12a starts 6 giving possible $10a+12a$ ranges 180-198 and 380-398 determining square=196, 10a starts 1 pre-entry. 10a/12a options are 127/69, 128/68 and 129/67. 7d pre-entry starts 37, 38 or 39 i.e. in the range 370-399. 3a entry is 695 or 795. The 7d calculations are:

3a	7d	3a-7d	Square
695	370-399	296-325	324
795	370-399	396-425	400

Either $12a=69, 7d=371$ entered as 771 with 3a entered as 695 or $12a=67, 7d=395$ entered as 695 with 3ac entered as 795.

7. $2d$ is a multiple of $6a$, $6a+2d$ is a square. $6a$ ends 6 or 7 giving $2a/6a$ candidates $64/16$, $34/17$, $54/27$, $74/37$ and $94/47$. Only $54+27$ adds to a square 81. $7d$ is entered as 771, $3a$ as 695, $12a=69$ completing the solution. Final table is:

3-digit entries	3a	8a	10a	13a	1d	4d	7d
Answer	2	6	1	4	9	7	3
Entry	6	4	2	3	1	9	7